

FLUCTUATION PROPAGATOR AND HEAVY QUARK DIFFUSION

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The quark fluctuation propagator is evaluated.
 It defines the diffusion coefficient in the vicinity of the phase transition
 and the gradient term in the Ginzburg-Landau functional.

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1. Introduction

Recent experiments performed at RHIC have revealed an unexpectedly large heavy flavor suppression. This result may indicate that the heavy quark diffusion coefficient is anomalously small in the vicinity of the critical line of the QCD phase diagram in the (μ, T) plane. The diffusion coefficient enters also into the Ginzburg-Landau functional via the term proportional to the $\langle A_\mu^2 \rangle$ condensate [1]. Comparison with the lattice calculations of $\langle A_\mu^2 \rangle$ shows that due to some dynamical reasons the diffusion coefficient is much smaller than the value given by the simple Drude formula [1]. It is well known that the diffusion coefficient may become small in the fluctuation regime and turns zero at the Anderson localization edge.

The transport coefficients are expressed in terms of the time-dependent propagator. Below we draw the fluctuation quark propagator (FQP) in its simplest form as a preliminary step in the investigation of the heavy quark dynamics in the vicinity of the phase transition. Our derivation closely follows the guidelines of the condensed matter physics [2].

Denoting the FQP as $L(\vec{p}, \omega)$ we may define it as

$$L^{-1}(\vec{p}, \omega) = -\frac{1}{g} + F(\vec{p}, \omega), \quad (1)$$

$$F(\vec{p}, \omega) = \sum_k G(\vec{k}, k_4) G(\vec{p} - \vec{k}, \omega - k_4), \quad (2)$$

Here g is the coupling constant with the dimension m^{-2} , the sum over k implies the momentum integration, Matsubara and discrete indices summation, $G(\vec{k}, k_4)$ in the thermal Green's function which reads

$$G(\vec{k}, k_4) = \frac{i}{\vec{\gamma}\vec{k} + \gamma_4 k_4 - im + i\mu\gamma_4}, \quad (3)$$

with $k_4 = -\pi(2n+1)T$, $T = \beta^{-1}$. We shall compute $F(\vec{p}, \omega)$ in the approximation of long-wave fluctuations

$$F(\vec{p}, \omega) \simeq A(\omega) + B(\omega)\vec{p}^2, \quad (4)$$

First we compute the term $A(\omega)$. We have

$$\text{tr}\{G(\vec{k}, k_4)G(-\vec{k}, \omega - k_4)\} = 2 \left\{ \frac{1}{\tilde{k}_4^2 + (\varepsilon - \tilde{\mu})^2} + \frac{1}{\tilde{k}_4^2 + (\varepsilon + \tilde{\mu})^2} \right\}, \quad (5)$$

where tr is over the Lorentz indices, $\varepsilon^2 = \vec{k}^2 + m^2$, $\tilde{k}_4 = k_4 - \omega/2$, $\tilde{\mu} = \mu - i\omega/2$. The second term in (5) corresponds to antiquarks. We shall omit it here only for brevity though as shown in [1] the interplay of the quark and antiquark modes may result in instability in the chiral limit. For the same reason we put $m = 0$. Performing the momentum integration around the Fermi surface we obtain

$$A(\omega) = \nu \sum_{n \geq 0} \frac{1}{\left(n + \frac{1}{2} + \frac{\omega}{4\pi T}\right)}, \quad (6)$$

where $\nu = \frac{2\mu^2}{\pi^2}$ is the density of states at the Fermi surface for two quark flavors. To evaluate $B(\omega)$ we act by the operator $\left(\vec{p} \frac{\partial}{\partial \vec{k}}\right)^2$ on the second Green's function in (2). The result reads

$$B = -\frac{\nu}{48\pi^2 T^2} \sum_{n \geq 0} \frac{1}{\left(n + \frac{1}{2} + \frac{\omega}{4\pi T}\right)^3}, \quad (7)$$

The logarithmic divergence of the sum in (6) may be removed by the introduction of the critical temperature T_c . First we regularize (6) by introducing the high-frequency cut-off $n_{max} = \frac{\Lambda}{2\pi T}$, $\Lambda \gg \omega$. Then

$$A = \nu \left\{ \psi \left(\frac{1}{2} + \frac{\omega}{4\pi T} + \frac{\Lambda}{2\pi T} \right) - \psi \left(\frac{1}{2} + \frac{\omega}{2\pi T} \right) \right\}, \quad (8)$$

where $\psi(z)$ is the logarithmic derivative of the Γ -function. Next we replace Λ by the critical temperature T_c using the relation

$$t = \ln \frac{T}{T_c} = \frac{1}{\nu g} - \psi \left(\frac{1}{2} + \frac{\Lambda}{2\pi T} \right) - \psi \left(\frac{1}{2} \right). \quad (9)$$

Now we can return to the underlying formula (1) for the FQP, collect all the pieces together and write

$$L^{-1}(\vec{p}, \omega) = -\nu \left\{ t + \psi \left(\frac{1}{2} + \frac{\omega}{4\pi T} \right) - \psi \left(\frac{1}{2} \right) - \frac{\vec{p}^2}{96\pi^2 T^2} \psi'' \left(\frac{1}{2} + \frac{\omega}{4\pi T} \right) \right\}, \quad (10)$$

Equation (10) is the basic one in condensed matter fluctuation theory [2]. We have shown that it can be almost literally retrieved within rather general approach to dense finite temperature quark matter. Fluctuations in quark matter are many orders of magnitude stronger than in ordinary and even in high temperature superconductors [1]. Quark matter formed in heavy ion collisions has a finite volume which also increases fluctuation effects. Close to the phase transition quark matter is a system with strong disorder [1] possibly revealing Anderson localization. The FQP is known to be an effective tool to study the properties of such systems. In particular the poles of $L(\vec{p}, \omega)$ determine the dynamical diffusion coefficient. The detailed investigation of this problem is beyond the scope of the present quick paper.

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